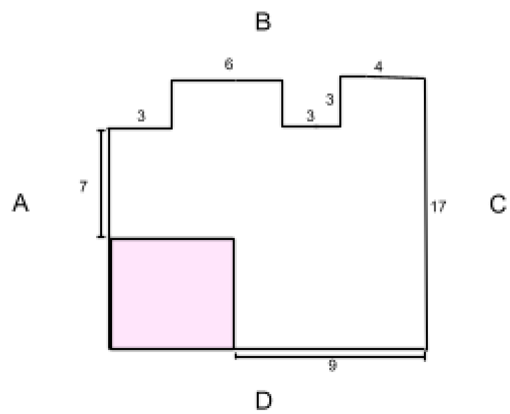
Pinkletes April Competition Solution 2024 - Geometry, Pascal's Triangle, Pigeon Hole Principle

1. Solve for the sum of the 5th row of pascal's triangle using powers of 2.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

The 5th row is 1 4 6 4 1 = 1 + 4 + 6 + 4 + 1 = 16 = 2^4

2. Calculate the area of the pink shaded region



Since side A=C and B=D, we can conclude that A=C=17 and 3+6+3+4=16=B=D. With this information we can first solve for the horizontal component of the pink figure.

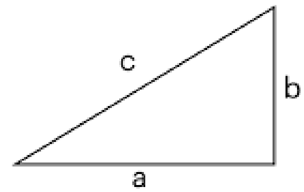
X+9 = 16 -> x=7

Next for the horizontal component,

7+3+y=17 -> y=7

The area is therefore equal to 7 x 7 = 49

3. Calculate side C using pythagorean's theorem, If side a=5 and b=12. *(c^2 = c x c, a^2 = a x a, b^2 = b x b)



$$c^2 = a^2 + b^2$$

Using pythagorean's theorem, c x c = a x a + b x b. Plugging our values in, we get c x c = 25 + 144 = 169. Now you can recognize that the square root of 169 is 13
c=13

4. Using Pascal's triangle, calculate the value of 11^6.

We can use Pascal's rule of powers of 11 and the 7th row
1 6 15 20 15 6 1

1 + 6(10) + 15(100) + 20(1000) + 15(10000) + 6(100000) + 1(1000000) = 1171561

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            1
          1   1
        1   2   1
      1   3   3   1
    1   4   6   4   1
  1   5   10  10  5   1
1   6   15  20  15  6   1
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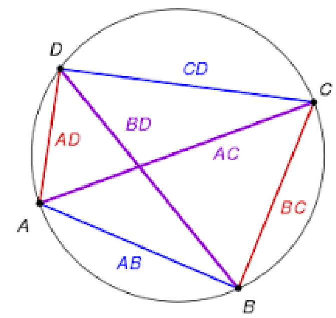
5. Suppose there are 7 chocolate chip cookies to be shared with 6 friends. Is it possible to guarantee that one friend will get two cookies?

Since there are n=7 chocolate chip cookies and n=7-1 number of friends. Each friend must get at least one cookie, which leaves us with one remaining cookie. Therefore, the answer is yes.

6. Using Ptolemy's theorem, If BD = 14, AD = 10, BC = 12, AB = 6, and CD =8 , what does AC equal?



Ptolemy's theorem states AC x BD = (AD x BC) + (CD x AB)
Plugging in the given values, we get the equation
AC x 14 = (10 x 12) + (6 x 8) = 120 + 48 = 168
AC = 12

7. Ben has 10 types of 3 flowers in a bouquet, each a different shade of pink (total of 30). How many flowers does he have to pull from the bouquet to guarantee that he has drawn 3 flowers of the same shade.

In the worst case, Ben will draw 2 flowers of each color, giving him 20. His 21st draw will guarantee that he has drawn 3 flowers of the same color

8. Use binomial expansion from pascal's triangle to expand $(a+b)^6$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

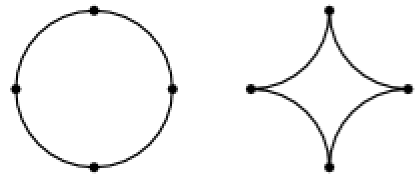
First, notice that we'll be using 1 6 15 20 15 6 1 as our coefficients. Next, we'll follow a pattern of decreasing the exponent of a and increasing the exponent of b from respectively 6 to 0 and 0 to 6.

$a^6 b^0 + a^5 b^1 + a^4 b^2 + a^3 b^3 + a^2 b^4 + a^1 b^5 + a^0 b^6$
$= a^6 + a^5 b + a^4 b^2 + a^3 b^3 + a^2 b^4 + a b^5 + b^6$

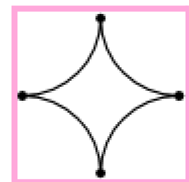
Now, we can insert our coefficients accordingly

$1a^6 + 6a^5 b^1 + 15a^4 b^2 + 20a^3 b^3 + 15a^2 b^4 + 6a^1 b^5 + 1b^6$

9. A circle of radius 1 is cut into four congruent arcs. The four arcs are joined to form the star figure shown. What is the ratio of the area of the star figure to the area of the original circle



Draw a square with side length 2 around the star figure (It has a side length of 2 because the diameter is equal to 2 times the radius). Now we can notice that the square forms 4 quarter circles of area $\pi/4$ each. Therefore the area of the star figure is the area of the square minus the 4 quarter circles.

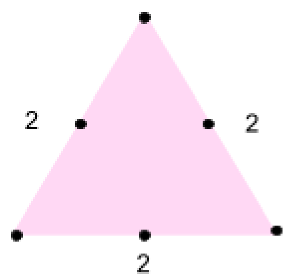


$2(2) - (\pi/4+\pi/4+\pi/4+\pi/4) = 4-\pi$

The area of the circle is π

$(4-\pi)/\pi$

10. Given four points inside an equilateral triangle of side length 2, prove that there are two points whose distance from each other is 1.



Looking at the equilateral triangle, you can notice that the original triangle splits into four equilateral triangles of side length 1.
By the Pigeonhole Principle, one of these four triangles must contain two of the four points. The maximum number of points you can choose without two points sharing a distance of 1 is three, which are the outer corners. If four points are chosen, then you are guaranteed to have two points that are 1 away from each other