

2023 Winter Pinkletes Mathematics Competition

Name: _____

Date: _____

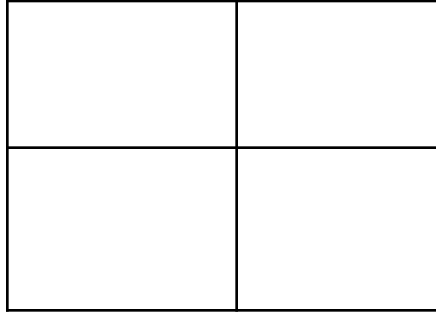
1. If $17|357$ and $357|6069$, does that mean $17|6069$? Explain.
($n|m$ means n is a factor of m)

2. Find the value of the expression: $7! / (4! * 3!)$
(For Example: $n! = 1 \times 2 \times \dots \times n$)

3. Sarah is planning her outfits for the week. She has 3 different tops, 4 different bottoms, and 2 different pairs of shoes. How many unique outfit combinations can she create for the entire week if she wears one top, one bottom, and one pair of shoes each day?

4. Prove 717607 is divisible by 11

5. A farmer wants to plant his 4 crops: corn, wheat, potatoes, and carrots. However in his 2x2 plot, he does not want corn and carrots to share a border or wheat and potatoes to share a border. How many combinations does he have to plant his crops if all plants must be planted?



6. If $a|246$ and $a|250$, then can we state $a|4$?
($n|m$ means n is a factor of m)

7. Sally wants to make a crazy yogurt combination using 5 different flavors of yogurt and 10 different toppings. If she wants each cup to hold 2 distinct flavors and 5 distinct toppings, how many different crazy yogurt combinations can she make?

8. For a positive integer n , the factorial notation $n!$ represents the product of the integers from n to 1. What value of N satisfies the following equation?

$$5! \cdot 9! = 12 \cdot N!$$

9. Three A's, three B's, and three C's are placed in the nine spaces so that each row and column contains one of each letter. If A is placed in the upper left corner, how many arrangements are possible? (2008 AMC 8A #14)

A		

10. What is the unit digit of the following sum? (Double factorial means you multiply every other number of a value n, Ex $10!! = 10 \times 8 \times 6 \times 4 \times 2$)

$$2!! + 4!! + 6!! + \dots + 2018!! + 2020!! + 2022!! + 2023!! + 2024!!$$

2023 Winter Pinkletes Mathematics Competition Solutions

1. If $17|357$ and $357|6069$, does that mean $17|6069$?

Yes, since 17 is a factor of 357 and 357 is a factor of 6069, then 17 must be a factor of 6069 by transitivity

Alternate solution: $17x = 357$, for some value of x

$$357y = 6069, \text{ for some value of } y$$

$$\text{By substitution, } 17xy = 6069$$

2. Find the value of the expression: $7! / (4! * 3!)$

$$(7 \times 6 \times 5 \times 4 \times 3 \times 2) / (4 \times 3 \times 2 \times 3 \times 2)$$

$$(7 \times 6 \times 5) / (3 \times 2)$$

$$35$$

3. Sarah is planning her outfits for the week. She has 3 different tops, 4 different bottoms, and 2 different pairs of shoes. How many unique outfit combinations can she create for the entire week if she wears one top, one bottom, and one pair of shoes each day?

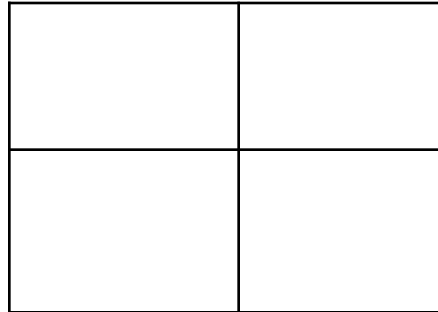
$$3 \times 4 \times 2 = 24$$

4. Prove 717607 is divisible by 11

Using the divisibility rule for 11, the given number can only be divided by 11 if the difference of the sum of digits at odd position and sum of digits at even position in a number is 0 or 11.

$$7 + 1 - 7 + 6 - 0 + 7 = 0$$

5. A farmer wants to plant his 4 crops: corn, wheat, potatoes, and carrots. However in his 2x2 plot, he does not want corn and carrots to share a border or wheat and potatoes to share a border. How many combinations does he have to plant his crops if all plants must be planted.



Since carrots and corn cannot share a border and wheat and potatoes cannot share a border, these two pairs must be diagonal to each other. There are 4 possible combinations to place carrots and corn.

Ca ___ co ___ ___ ca ___ co
 ___ Co ___ ca co ___ ca ___

In each of these four combinations, there are only two ways to plant wheat and potatoes. Therefore, $4 \times 2 = 8$.

6. If $a|246$ and $a|250$, then can we state $a|4$?

Yes, following the law of divisibility that states

if $a|b$ and $a|(b \pm c)$, then $a|c$

7. Sally wants to make a crazy yogurt combination using 5 different flavors of yogurt and 10 different toppings. If she wants each cup to hold 2 distinct flavors and 5 distinct toppings, how many different crazy yogurt combinations can she make?

$5C_2$ (5 choose 2) ways to select 2 flavors of yogurt $\rightarrow 5!/((5-2)!2!) = 10$

$10C_5$ (10 choose 5) ways to select 5 toppings $\rightarrow 10!/((10-5)!5!) = 252$

Answer: $252 \times 10 = 2520$

8. For a positive integer n , the factorial notation $n!$ represents the product of the integers from n to 1. What value of N satisfies the following equation?

$$5! \cdot 9! = 12 \cdot N!$$

On the left side of the equation we can rewrite $5! \cdot 9!$ into $(5 \times 4 \times 3 \times 2 \times 1 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1)$. Next, we multiply $4 \times 3 = 12$ and are left with $9! \times 5 \times 2$. Notice that $5 \times 2 = 10$, therefore, $5! \cdot 9!$ can be rewritten as $12 \times 10!$

Answer: 10

9. Three A's, three B's, and three C's are placed in the nine spaces so that each row and column contains one of each letter. If A is placed in the upper left corner, how many arrangements are possible? (2008 AMC 8A #14)

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There are 2 ways to place the remaining A's, 2 ways to place the remaining B's, and 1 way to place the remaining C's for a total of $(2)(2)(1) = \boxed{(C) 4}$.

Solution from Art of Problem Solving

10. What is the unit digit of the following sum? (Double factorial means you multiply every other number of a value n, Ex $10!! = 10 \times 8 \times 6 \times 4 \times 2$)

$$2!! + 4!! + 6!! + \dots + 2018!! + 2020!! + 2022!! + 2023!! + 2024!!$$

First, we should notice that any value of n that is greater than or equal to 10 will have a unit digit of 0 because it has a factor of 10. Therefore, we only need to find the unit digit of $2!!$, $4!!$, $6!!$, and $8!!$

$$2!! = 2$$

$$4!! = 8$$

$$6!! = 48$$

$$8!! = 384$$

$$2 + 8 + 8 + 4 = 22 \rightarrow 2$$